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BIALGEBRAS IN LOCALLY PRESENTABLE CATEGORIES

by

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Preface

This paper constitutes a second part to "Lokal präsentierbare Kategorien" by P.Gabriel and the author (Lecture Notes vol. 221). The reader need only be familiar with the basic facts about locally presentable categories. The relevant material is collected in § 2.

The starting point was marked by unsolved problems in the first part. They were successfully tackled step by step in [31], [32], [33] and [34]. In the process the notion of a bialgebra - generalizing bialgebras over a commutative ring - emerged as a unifying concept. We give here a systematic treatment of bialgebras in locally presentable categories and then apply the results to the above mentioned problems and to problems in other areas as well, in particular to Hopfalgebras, bialgebras, coalgebras over a commutative ring and to descent data, etc.

This material was first presented in seminars of H.Schubert in the summer semesters 1975 and 1976. I profited a great deal from the lively discussions with the participants. I also would like to thank A.Kock and W.Wischnewsky for discussions in Amiens about Hopfalgebras, bialgebras, coalgebras, comodules etc. over a commutative ring. Without their disbelieve I would not have tried to prove that these categories are locally presentable. I am indebted to M.Barr and T.Fox for discussions later on in Zurich about problems associated with Props. This put me on the track to look for something better (namely bialgebras). Finally M.Tierney raised at the AMS meeting in Toronto the question of the relationship between bialgebras and sections (resp. cartesian closed sections) of a fibration. This turned out to be very fruitful.

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tarial positions. His ingenuity in making life difficult at a universi-
ty with plenty of jobs and funds is only exceeded by his endurance in
doing so.

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